Ultra-refraction phenomena in Bragg mirrors

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Abstract

We show numerically for the first time that ultra-refractive phenomena do exist in one-dimensional photonic crystals: we exhibit the main features of ultra-refraction, that is the enlargement and the splitting of an incident beam. We give a very simple explanation of these phenomena in terms of the photonic band structure of these media.

It has recently been shown numerically as well as experimentally that near a band edge, photonic crystals could behave as if they had an effective permittivity close to zero [1–3]. Such a property induces unexpected behaviors of light usually called ultra-refractive optics. The main phenomena are the splitting or the enlargment of an incident beam, or a negative Goos-Hänschen effect [4]. The common explanation of these facts lie on the study of the photonic dispersion curves. Though appealing, it seems difficult to turn this explanation into a rigorous one as the notion of group velocity in a strongly scattering media seems doubtful apart in the homogenization sense which is not the situation for ultrarefractive optics. In our opinion, these surprising and beautiful phenomena mainly rely on the rapid change in the behavior of the field inside the structure when crossing a band edge. In this article, we provide a rather simple explanation of some of these phenomena (splitting and enlargment of an incident beam), which implies that they should be observed with one dimensional structures (as foreseen by [1]). Indeed, we show by numerical experiments that it is the case in Bragg mirors (the simplest photonic crystals).

From a theoretical point of view, we consider a periodic one dimensional medium characterized by its relative permittivity $\varepsilon(x)$, which is assumed to be real, illuminated by a plane wave. It is well known that the band structure is determined by the monodromy matrix $\mathbf{T}$ of one layer [5,6], that is, the matrix linking the field and its derivative over one period. This matrix is a function of λ and θ. The main quantity is then $\phi(\lambda,\theta) = \frac{1}{2} tr(\mathbf{T}(\lambda,\theta))$. When $|\phi(\lambda,\theta)|$ is inferior to 1 then (λ,θ) belong to a conduction band, and when $|\phi(\lambda,\theta)|$

is superior to 1 then (λ, θ) belong to a gap. In fig. 1 we give a numerical example for a Bragg Mirror with $\varepsilon_1 = 1$, $\varepsilon_2 = 4$, $h_1 = h_2 = 1$ (the lengths are given in λ units).

Now let us use a Gaussian beam as the incident field. Let us suppose that the mean angle of the beam is zero (normal incidence) and that its wavelength is very near a band edge. Then two things may happen. Reasoning on the oriented wavelengths axis, if the beam is centered on the left side of the gap (the dispersion diagram is given in the plane (λ, θ), if one uses frequencies instead of wavelengths one has to exchange left and right), the center of the beam belongs to a conduction band and the edges of the beam belong to the gap. Consequently, after propagation in the medium, the transmitted field has a narrowed spectral profile, and therefore the beam is spatially enlarged (figures 1,2). Conservely, if the beam is centered on the right side of the gap, then the center of the beam belongs to the gap, and the edges of the beam belong to the conduction band. Therefore, the transmitted field has two well separated peaks and the beam is splitted in two parts (figures 1,3). The fundamental remark here is that ultra-refractive phenomena are due to the rapid variation of the conduction band with respect to the angle of incidence, *in complete contradiction with the habitual requested properties of photonic crystals*, which are expected to have a dispersion diagram quite independent of the angle of incidence.

Let us now check numerically the above explanations. We still use the previous Bragg Mirror. The numerical experiments are done with an s-polarized incident field of the form:

$$u^i(x, y) = \int A(\alpha) \exp(i\alpha x - i\beta(\alpha) y) \, d\alpha \tag{1}$$

with $\alpha = k \sin\theta, \alpha = k \sin\theta_0, \beta(\alpha) = \sqrt{k^2 - \alpha^2}$ and $k = 2\pi/\lambda$, $|A(\alpha)| = \dfrac{W}{2\sqrt{\pi}} \exp\left(-\dfrac{(\alpha - \alpha_0)^2 W^2}{4}\right)$. In all numerical experiments $W = 0.5$, the variable θ_0 is the mean angle of incidence.

In the first numerical experiment, we set $\lambda = 2.7$ and $\theta_0 = 0°$. We have plotted in figure (4a) the transmission coefficient as well as the spectral profile of the transmitted beam. Obviously, this profile is much narrower than the incident one. The map of the electric field is given in figure (4b). The incident field is coming from below. As expected, we observe a strong enlargement of the transmitted beam.

For the second numerical experiment, we use $\lambda = 3$ and $\theta_0 = 0°$. This time, the center of the beam belong to the gap. We have plotted in figure (5a) the transmission coefficient as well as the spectral profile of the incident and transmitted fields. It appears that there are two isolated peaks, and therefore the transmitted field is splitted spatially into two parts, as shown in figure (5b). At that point it is easily seen that by switching the incident beam it is possible to keep only one transmitted beam. This is done in the last experiment, where we set $\theta_0 = 10°$. As it can been seen on fig 6 (a), only the right part of the beam is significantly transmitted, and thus there is only one transmitted beam (fig. 6 (b)). If Snell-Descartes law is directly applied to this situation, then it seems that the medium has an optical index that is inferior to 1.

As a conclusion, we have shown both theoretically and numerically that ultra-refractive phenomena do happen in one-dimensional Bragg mirrors, or more generally in one dimensional photonic crystals. They may be well explained by means of the intersection of the support of the incident beam with the gaps and the conduction bands. It must also be

noted that, though one dimensional photonic crystals exhibit ultra-refractive properties, bidimensional or three dimensional ones should show a better efficiency due their richer band diagrams. Nevertheless, doping 1-D structure or using quasi-crystals may enable a fair control over the width of the gaps and conduction bands, thus leading to the design of practical devices. Finally, it should also be noted that such a surprising phenomenon as a negative Goos-Hänchen effect does not seem to be possible in 1D structures.

Figure captions:

figure 1: Dispersion diagram of a Bragg mirror, with $\varepsilon_1 = 1, \varepsilon_2 = 4, h_1 = 1, h_2 = 1$. The double arrowed lines indicate the width of the Gaussian beams.

figure 2: Sketch of the behavior of the beam when spatially enlarged.

figure 3: Sketch of the behavior of the beam when splitted.

figure 4: (a) Transmission through the Bragg mirror vs. angle of incidence (dotted line), spectral amplitude of the incident beam (solid line) and spectral amplitude of the transmitted beam (thick line) ($\lambda = 2.7, \theta_0 = 0$).

(b) Map of the intensity of the electric field above and below the Bragg mirror in the case of figure 2 (above: transmitted field, below: incident field).

figure 5: (a) same as fig. 4 (a) in the case of figure 3 ($\lambda = 3, \theta_0 = 0°$).

(b) Map of the intensity of the electric field above and below the Bragg mirror in the case of figure 3 (above: transmitted field, below: incident field).

figure 6: (a) same as fig. 4 (a) in the case of figure 3.($\lambda = 3, \theta_0 = 10°$).

(b) Map of the intensity of the electric field above and below the Bragg mirror in the case of figure 3 (above: transmitted field, below: incident field).

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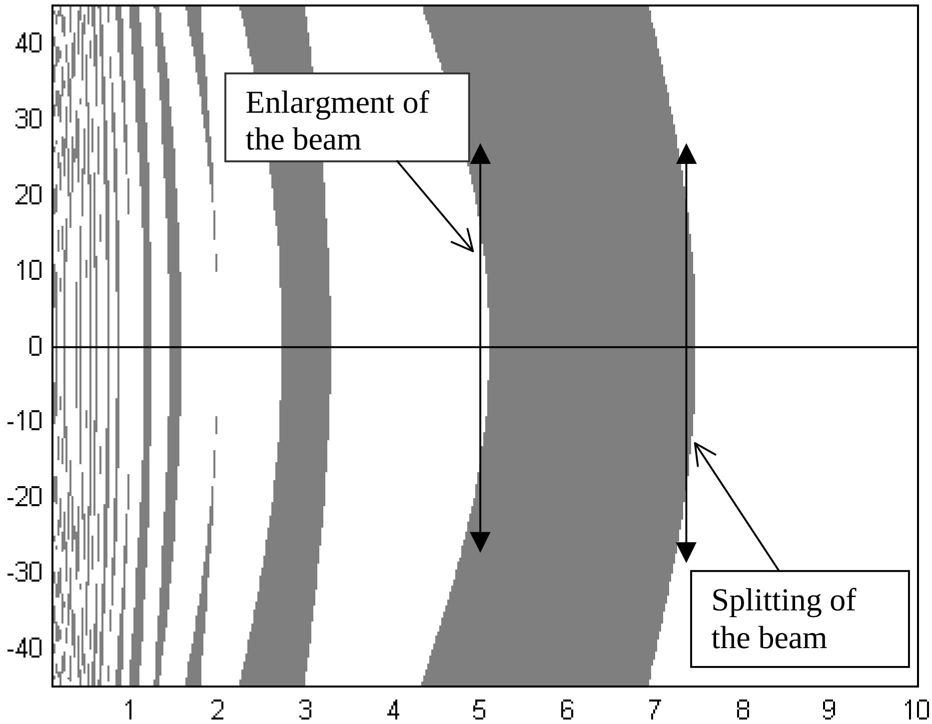
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30
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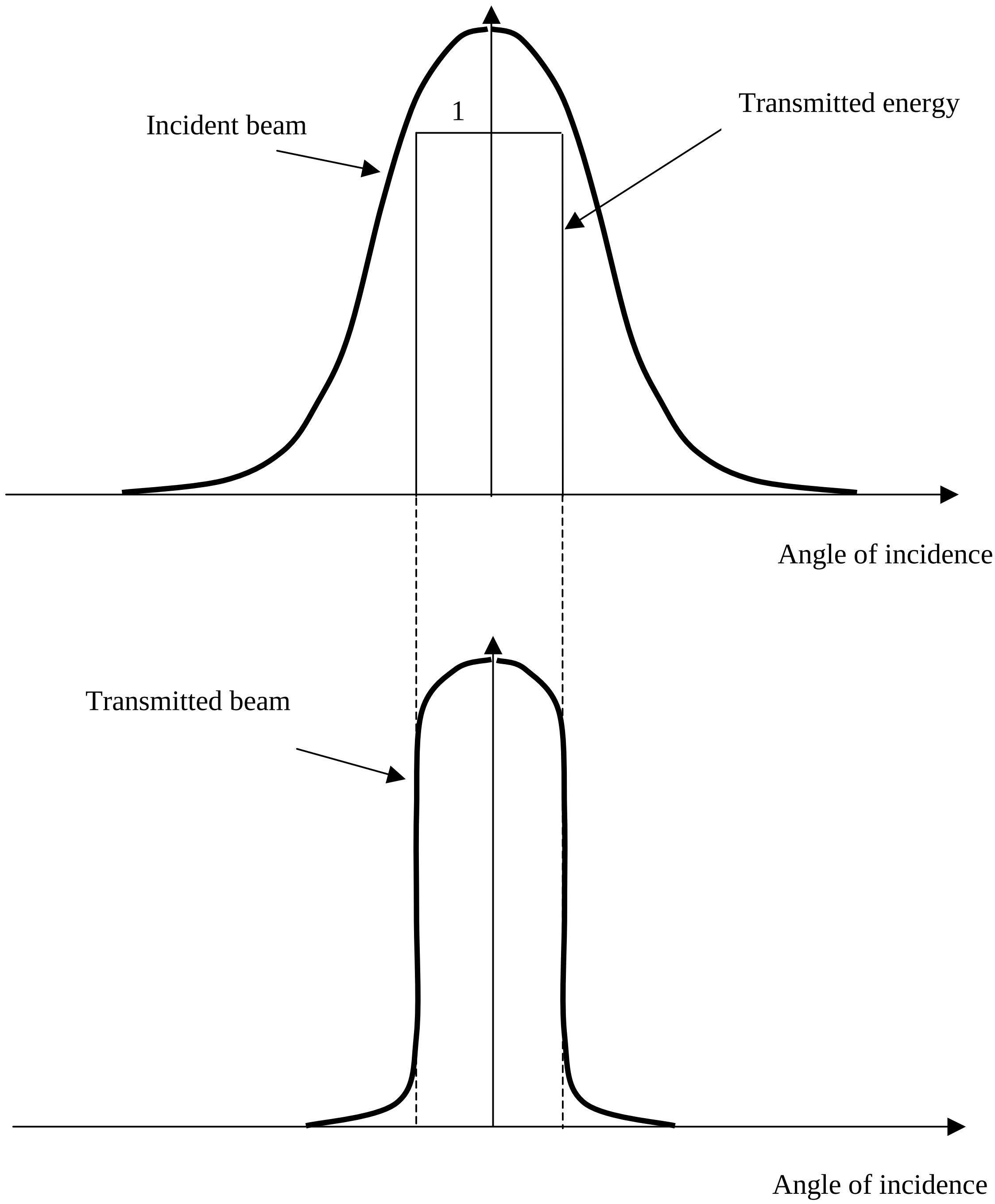
Enlargment of the beam
Splitting of the beam

Figure 1



Incident beam
1
Transmitted energy
Angle of incidence
Transmitted beam
Angle of incidence

Figure 2

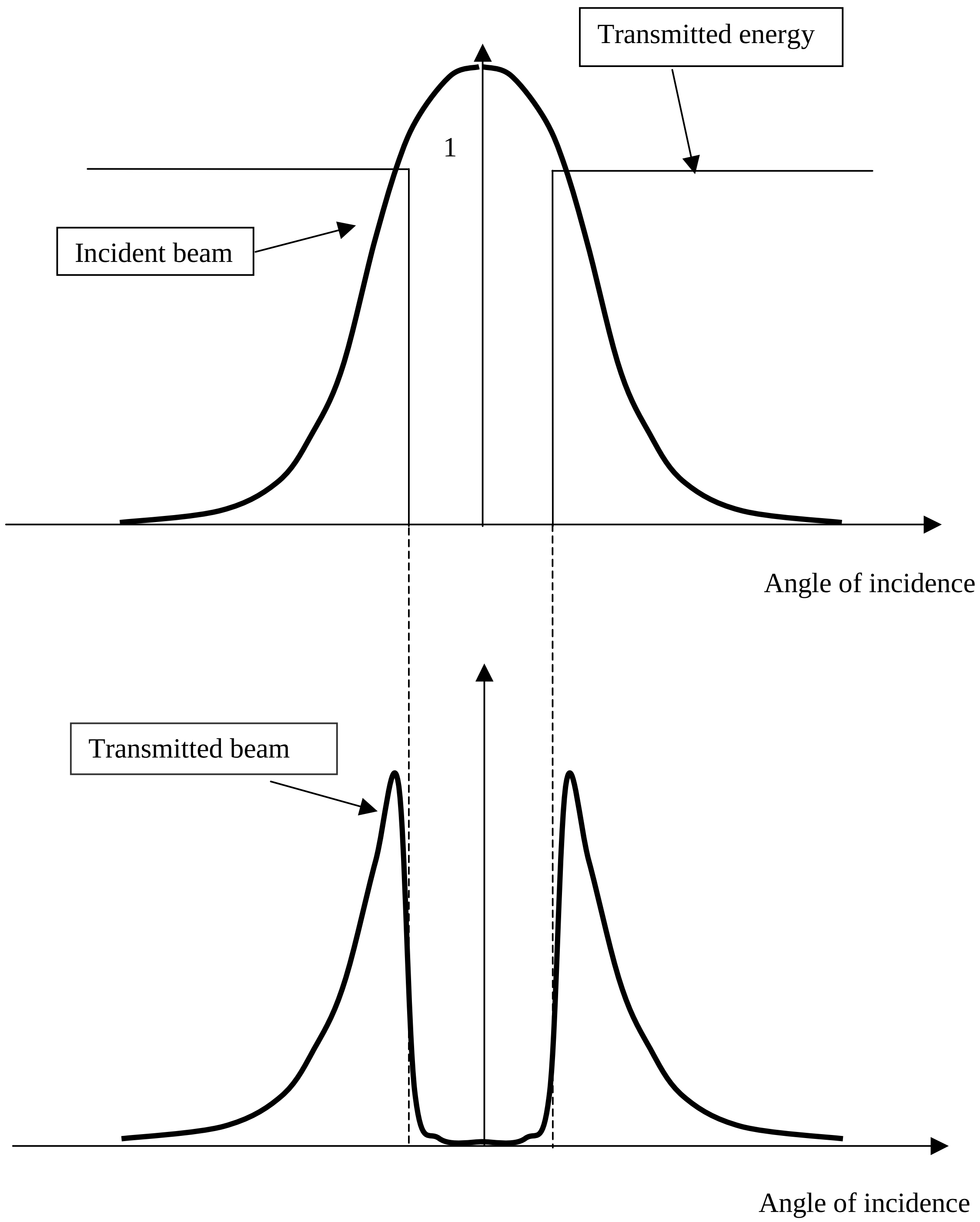
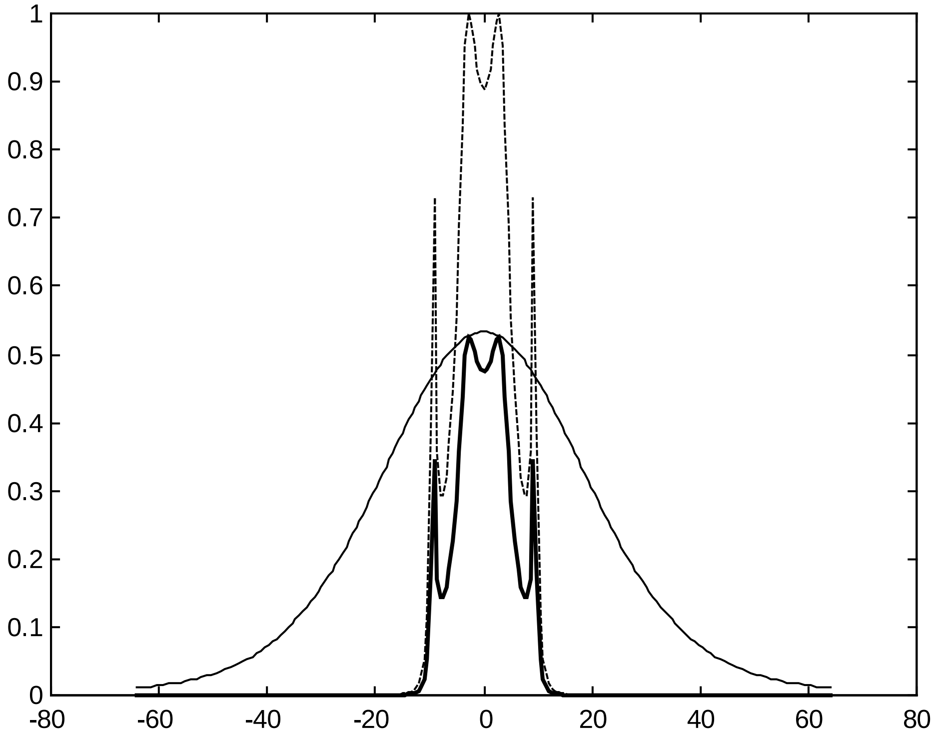

Transmitted energy
1
Incident beam
Angle of incidence
Transmitted beam
Angle of incidence

Figure 3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0.6
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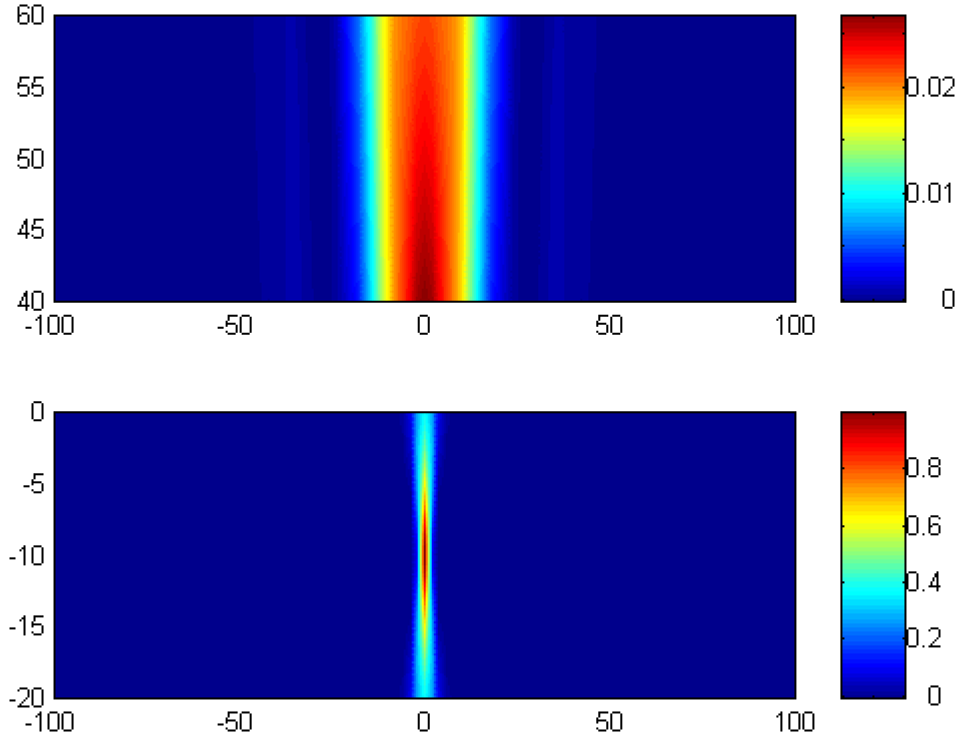
0.3
0.2
0.1
0
-80
-60
-40
-20
0
20
40
60
80

(a)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100
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0.01
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0
-5
-10
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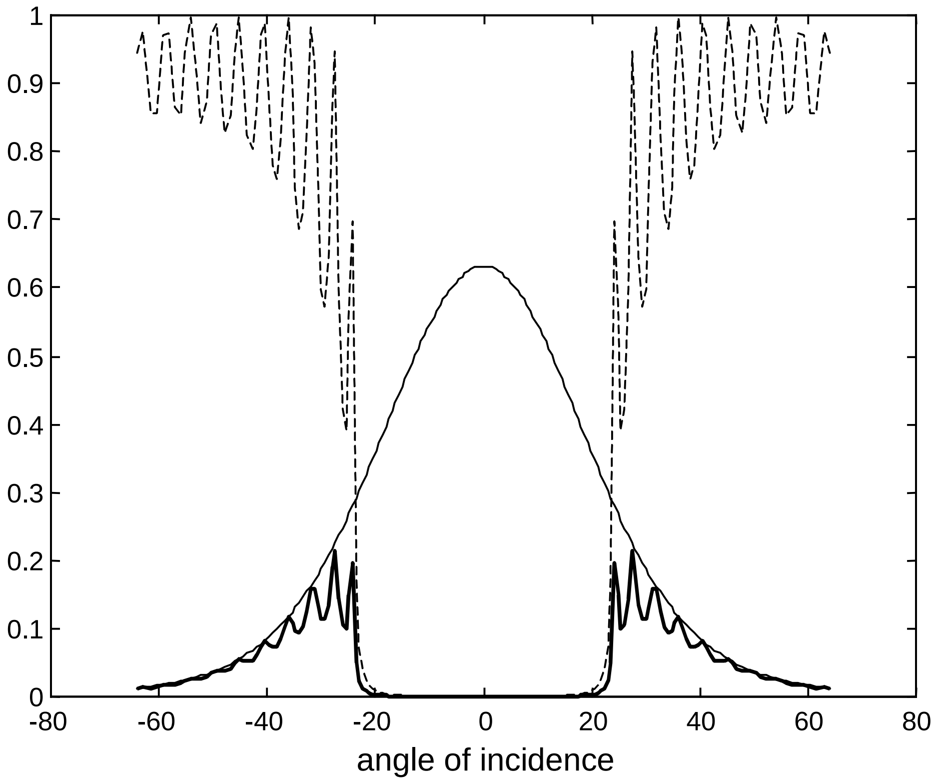
-20
-100
-50
0
50
100
0.8
0.6
0.4
0.2
0

(b)

Figure 4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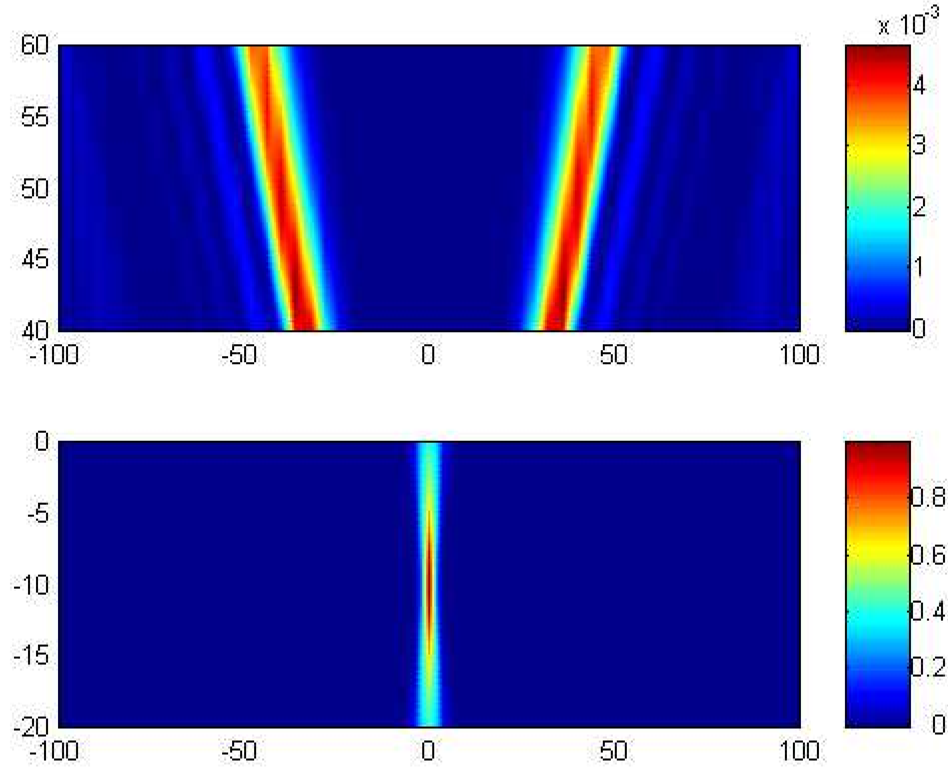
0
20
40
60
80
angle of incidence

(a)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-100
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0
-5
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-100
-50
0
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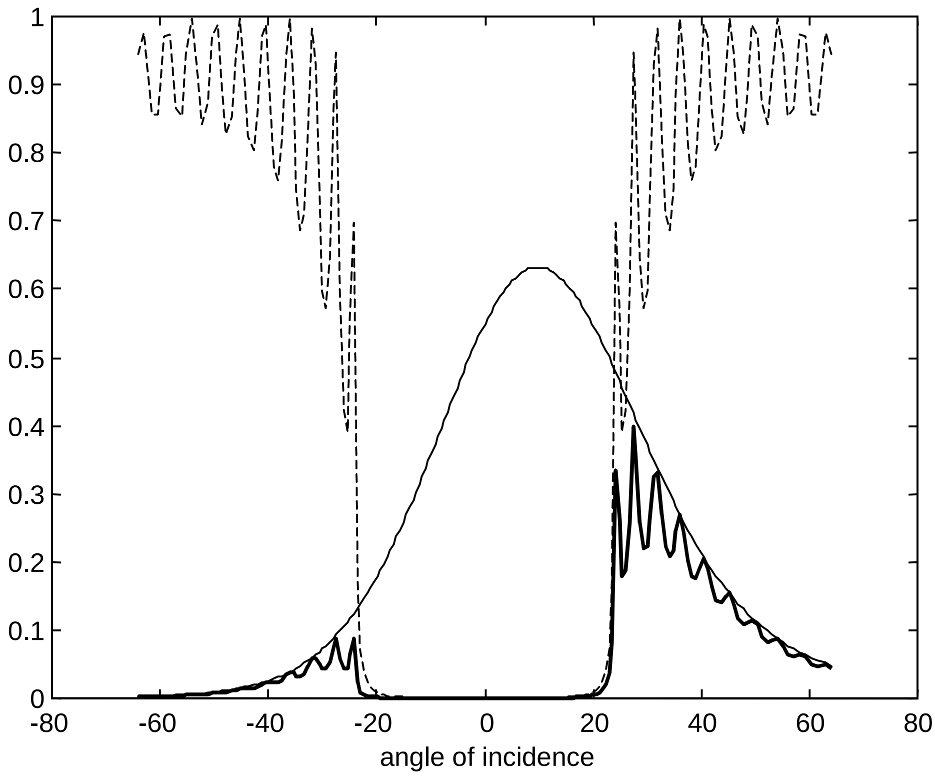
0.4
0.2
0

(b)

Figure 5



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-40
-20
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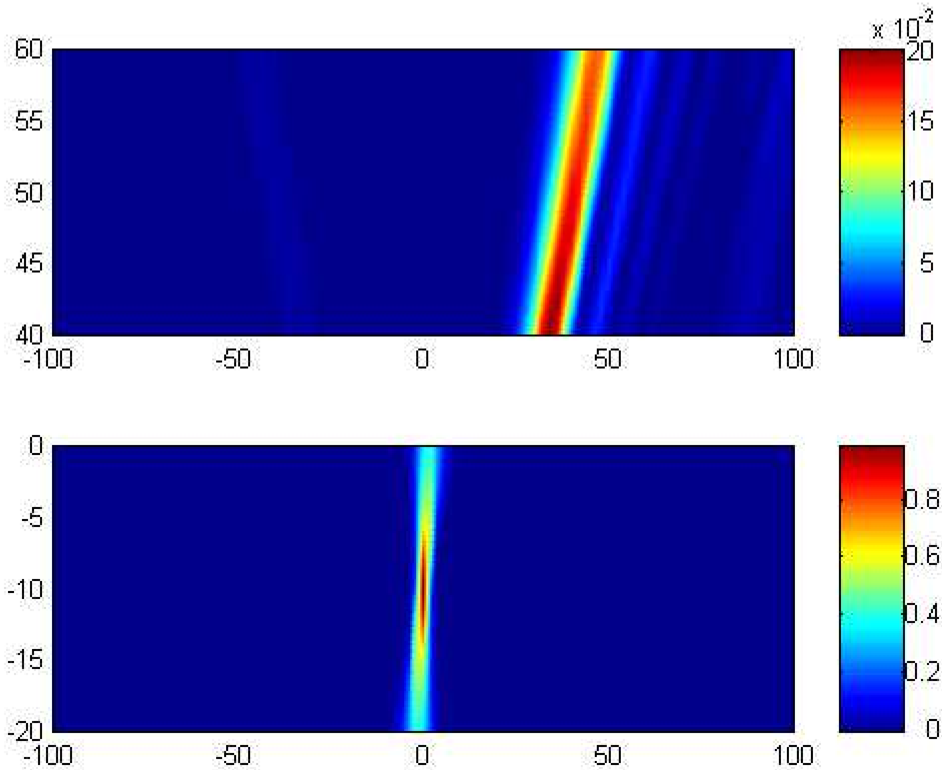
20
40
60
80
angle of incidence

(a)



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-5
-10
-15
-20
-100
-50
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100
0.8
0.6
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0

(b)

Figure 6